Generalized op-amp model simplifies analysis of complex feedback schemes

Jerrod Graeme, Burr-Brown Corp

When attempting to find a path through a forest of complex op-amp feedback arrangements, you can lose your way amidst all the branches on the trees. Consolidating the branches into standard blocks for a generalized op-amp model can help you forge a simple analysis path.

Feedback configurations for op amps having multiple inputs can be as convoluted as a labyrinth. Many complex feedback schemes don't conform to simple models and often pose difficult analysis challenges. A generalized op-amp model based on voltage-divider ratios can standardize analysis to cover a variety of feedback and input configurations. By making your circuit conform to this model, you can systematically determine its gain, bandwidth, frequency stability, and errors. Three circuits that illustrate the model's utility are a modified difference amplifier, an op amp with a multiplier in its feedback loop, and an op amp with a multiplier in its feedforward path—a voltage-controlled lowpass filter.

Fig 1 shows the generalized model. The model represents the op amp and all possible feedback and input connections to it. It consolidates the sum and difference operations associated with the op amp's two inputs, \( e_1 \) and \( e_2 \), into a single summing junction, \( \zeta \). (Note that the net op-amp input is \( e_i \).) Composite elements for positive and negative feedback are consolidated in blocks \( \alpha_+ \) and \( \alpha_- \), respectively. Similarly, the composite elements that represent the attenuation for positive and negative inputs to the op amp are consolidated in blocks \( \beta_+ \) and \( \beta_- \). Block \( A \) represents the idealized op amp's gain, and \( e_0 \) is the op amp's output.

The closed-loop gain (\( A_{\text{CL}} \)) of the generalized model is

\[
A_{\text{CL}} = \frac{e_0}{e_i} = \frac{\alpha/\beta}{1 + 1/\alpha \beta} = \frac{A \alpha_+}{1 + 1/\alpha \beta} \tag{1}
\]

where \( \beta = \beta_+ - \beta_- \) and \( \alpha = \alpha_+ - \alpha_- \) is the net amount of feedback for the model, and \( \alpha \) and \( \beta \) are the net feedback attenuation for the inputs to the op amp (see table in Fig 1). The numerator \( \alpha/\beta \) is the ideal closed-loop gain (\( A_{\text{CL}} \)) when the open-loop gain (\( A \beta \)) is much larger than 1. The denominator of the equation, \( 1 + 1/\alpha \beta \), is an expression that lets you determine the circuit's closed-loop bandwidth.

The model's closed-loop bandwidth is \( \text{BW} = \beta f_c \), where \( f_c \) is the unity-gain crossover frequency of the op amp's forward gain (\( A \)). The generalized model lets you determine the gain for any error signals present at the op amp's input terminals. All op amps amplify these error signals by a gain of

\[
A_{\text{CL,e}} = \frac{1/\beta}{1 + 1/A \beta}
\]
You can apply this closed-loop error gain to any of the op amp's error signals, such as the input-offset voltage, input-noise voltage, and common-mode-rejection voltage.

The Fig 1 model can accommodate one input to the op amp's positive input via the α block and one input to the negative input via the β block. For multiple inputs, you must add α blocks. You determine the transfer function for each α block using superposition principles.

To illustrate the utility of the generalized model, we'll analyze some op-amp circuits having complex feedback and input configurations. Fig 2a depicts a configuration where the gain of an added op amp (IC₂) modifies the feedback of difference amplifier IC₁. Without the added op amp, the equal-valued resistors (R) set the difference-amplifier gain to 1. The fixed resistor values maximize the common-mode rejection ratio, and factory adjustments can trim the resistor ratios to within 0.002%.

IC₂ lets you change the circuit gain without affecting the difference-amplifier's common-mode rejection ratio. You could analyze this circuit using A₁, A₂, and R blocks for each amplifier. Using the generalized model, however, you can simplify analysis by including IC₂ in the β₁ feedback path for IC₁. The fraction of the output voltage (eₒ) fed back to IC₁'s noninverting input is β₁ = -R₂/R₁. Because there is no feedback to IC₁'s inverting input, β₂ = 0. The net feedback factor is β = β₁ - β₂ = R₂/2R₁.

The equal values for the resistors surrounding the difference amplifier determine α₁ = α₂ = ½. In addition, the differential input signal is eᵁ = eₒ - eᵢ. Substituting eᵁ, α₁, and β into the equation A_cl₂ = eₒ/eᵁ = α/β yields

where eₒ is the ideal output signal. For the resistor values in Fig 2a, the addition of op amp IC₂ modifies the difference amplifier's ideal closed-loop gain (A_cl₂) from 1 to 10.

You can analyze the circuit's frequency stability by examining the frequency-response curves for IC₁'s open-loop gain, |A₁|, and the inverse of the net feedback factor, 1/β₁. (Fig 2b). The two curves intersect at frequency BW. For Fig 2b, the rate of closure—the difference in the slopes of the |A₁| and 1/β₁ curves where they intersect—is 20 dB/decade. This rate implies a 90° phase margin and excellent overall frequency stability.

The unity-gain crossover frequency for IC₂ is f₂c and IC₂'s net feedback factor is β₂ = R₂/(R₁ + R₂). The rise in the circuit's 1/β₁ curve at f₂c shows the effect of IC₂'s closed-loop bandwidth on stability. When f₂c = 10BW, the rise in the 1/β₁ curve has negligible effect on stability. The typical crossover frequencies for the INA105 (IC₁) and OPA827 (IC₂) amplifiers are 2 and 16 MHz, respectively. Using β₁ = 0.05 and β₂ = 0.91, BW = β₂f₂ = 100 kHz, and BW = 4.6 MHz. The effect of IC₂'s closed-loop bandwidth on stability is negligible.

Two op amps can be better than one

You can also readily analyze the composite amplifier combination in Fig 3a using the generalized model. In this example, A_cl₂ = (1 + R₁/R₂) is the closed-loop gain of IC₂, and A₁ is the open-loop gain of IC₁. The second amplifier modifies gain-block A of the generalized model so that A = A₁A_cl₂. The transfer function for the A block has two poles (Fig 3b). One pole is due to the dominant pole for amplifier IC₁; the other pole is due to the closed-loop response of IC₂—BW₂ = β₂f₂. For frequency stability, the BW₂ pole must...
be considerably higher in frequency than the intercept point of the $|A_\beta|$ response curve and the 1/β response curve. For the circuit, $1/\beta = 1 + R_2/R_1$.

Introducing the second op amp reduces the error signals and increases the gain-bandwidth product of the circuit. The increase in loop gain due to $IC_2$ proportionally reduces the output error signal, $e_o/\Delta$, caused by any error signal at $IC_1$'s input terminals. The loop gain is the gain difference between the $|A|$ and 1/β curves. In addition, adding $IC_2$ increases the circuit's bandwidth from $BW_1$ to $BW_2$. For the resistor values in Fig 3a, the bandwidth increases by a factor of 10.

The generalized op-amp model also helps you analyze circuits having variable feedback elements. Variable feedback elements can include potentiometers, switches, and analog multipliers, all of which affect the magnitude and phase responses for 1/β. The circuit in Fig 4a places an analog multiplier in the feedback loop of an op amp to create an analog divider. The divider's transfer function is $e_o = 10R_2e_1/(R_1 + R_2)$. To employ the generalized model, put the gain of the multiplier in the β block. Without the multiplier, the feedback factor for op amp $IC_1$ is $R_1/(R_1 + R_2)$. The multiplier's transfer function, $XY/10$, scales $e_o$ by $e_2/10$. Therefore, the net feedback factor for the model, which takes into account $e_o$, is

$$\beta = \frac{e_o R_1}{10 (R_1 + R_2)}.$$

Fig 4b illustrates the effect of the $e_o$ variable control signal on the circuit's bandwidth and stability. The control-signal voltage moves the intercept point for the 1/β curve over the full range of the $|A|$ curve. When $e_o$ is 0V, β is zero, and the op amp operates open loop. When $e_o$ is 10V, the multiplier gain is 1,

![Fig 3](image1.png)  
**Fig 3**—You can increase the net open-loop gain and closed-loop bandwidth by placing an additional op amp in the forward gain path (a). The closed-loop response of the additional amplifier places a second pole in the A gain block (b), which determines the circuit's stability.

![Fig 4](image2.png)  
**Fig 4**—The addition of a multiplier in the feedback of an op amp lets you divide inputs from two signal sources (a). The net feedback factor $\beta$ varies with control voltage $e_2$ (b), so you must analyze the circuit's stability over the entire control-voltage range.
and the device can be replaced with a short circuit. Because the intercept point varies, you must pay close attention to the rate of closure of the two curves over the entire control-voltage range.

A pole in the multiplier's frequency response causes the 1/β curve to rise, as Fig 4b shows. To maintain circuit stability, the op amp must have a dominant pole so the moving intercept point always occurs on the flat portion of the 1/β curve. The OPA177 op amp's 0.6-MHz bandwidth and the MPY634 multiplier's 16-MHz bandwidth make the devices good choices to ensure the circuit's frequency stability. For the component values in Fig 4a, 1/β varies from 0.00091 to 0.091 as e2 varies from 0.1V to 10V. The circuit bandwidth changes proportionally from 560 Hz to 55 kHz.

To use the generalized model for circuits having multiple inputs to the op amp, you must apply superposition principles. Fig 5a shows an example where three input signals connect to the inverting input of an op amp through different networks. Figs 5b and 5c show redrawn versions of the circuit that let you use superposition to determine the closed-loop gain for each input source. The ideal closed-loop gains for each input source are set at ground: $\alpha_1 = 0.9$. Using the same approach to calculate the $\alpha_0$ for the input sources at $e_2$ and $e_3$ yields $\alpha_0 = 0.09$ and $\alpha_3 = 0.009$.

Although the attenuation (α) block for each input source is different, the circuit's bandwidth and stability considerations are the same for all input sources. The reason is that the denominator for $A_{CL}$ in Eq 1, which is $1 + 1/\beta$, is the same for all input sources and doesn't depend on α. The circuit's bandwidth is $BW = \beta_0$ for all input sources.

Increasing the number of op-amp inputs constrains the bandwidth of the circuit, however. For the Fig 5 example, if $e_3$ is the only input source, the net feedback factor is $\beta = R_2/(R_1 + R_2) = 0.5$. The OPA111 has a unity-gain cross-over frequency ($f_c$) of 2 MHz, so the circuit bandwidth is $BW = 0.5 \times 2 MHz = 1 MHz$. The addition of the other two input sources to the circuit reduces the net feedback factor to $\beta = 0.009$. Therefore, the added input sources decrease the circuit bandwidth to 18 kHz—a factor of 55.

The different α blocks determine different ideal closed-loop gains for each input source. The ideal closed-loop gain expression is $A_{CL} = \alpha/\beta$. Therefore, the ideal closed-loop gain for the input sources at $e_1$, $e_2$, and $e_3$ are $-100$, $-1$, and $-1$, respectively. However, the closed-loop bandwidth is still 18 kHz for all three input sources.

Finally, look at an op-amp configuration that employs multiple feedback paths. Fig 6 depicts a circuit that has feedback from the op amp's output and from the output of an added multiplier. The multiplier is a gain block in series with the circuit's forward gain path and affects the net forward gain as well as the net feedback factor. The multiplier and the resistor feedback convert the circuit from an integrator to a voltage-controlled lowpass filter.

The multiplier's transfer function is $XY/10$. When
Adding a multiplier in the feedforward path changes a basic integrator circuit into a voltage-controlled lowpass filter. This circuit shows how you can use the generalized model to analyze an op-amp configuration that employs multiple feedback paths.

Control-voltage $e_C$ is 10V, the multiplier acts as a short circuit. Under this condition, resistor $R_2$ and capacitor $C$ feedback produce a standard lowpass filter having a pole frequency, $f_p$, at $1/2\pi R_2C$. When $e_C$ is less than 10V, the attenuation to the voltage at pin X on the multiplier is $e_C/10$. The attenuation reduces the voltage driving $R_2$, which in turn reduces the current feedback to the op amp's summing junction. The effect is the same as increasing $R_2$'s value to $10R_2/e_C$. The increase in the effective resistance changes the lowpass-filter pole frequency to $e_C/20\pi R_2C$.

To analyze this circuit's performance using Eq 1 for the generalized model, you must determine $A_{CL}$, $A$, and $\beta$. At very low frequencies, the closed-loop gain is $-R_2/R_1$, because the capacitor has no effect. At higher frequencies, the voltage-controlled corner frequency determines the closed-loop roll-off. Therefore, the closed-loop gain is

$$A_{CL} = \frac{e_C}{e_i} = \frac{A}{1 + 1/\beta B} = \frac{-R_2/R_1}{1 + s(10R_2/e_0)C}$$

Both the op amp and the multiplier determine the forward gain of the model, so $A = A_{CL}/10$. The net feedback factor, $\beta$, depends on two feedback paths. Fig 7a depicts the feedback model for the Fig 6 circuit. The $P_C$ and $P_R$ blocks represent the feedback to the op amp's inverting input through the capacitor and resistor $(R_2)$, respectively. The net feedback factor is

$$\beta = (10/e_0)\beta_C + \beta_R.$$  

The voltage fed back through the capacitor precedes the attenuation caused by the multiplier. Therefore, $\beta_C$ is magnified by $10/e_0$, because the voltage at the multiplier's input is $(10/e_0)e_C$. Fig 7b shows the $1/\beta$ and $|A|$ curves for the Fig 6 circuit. The dashed curves show how the two curves change with variations in $e_C$. The $1/\beta$ and $|A|$ curves move up and down.
GeneraZed Op-Amp Model

together, so the intercept point and corresponding bandwidth remain fixed. The rise in the 1/f curve at high frequencies is due to the bandwidth of the multiplier and determines the rate of closure for the two curves.

The circuit demonstrates a condition not usually found in op-amp configurations. When the multiplier provides attenuation, the feedback is greater than 1 at high frequencies. Generally, the feedback cannot exceed 1, but in this case the multiplier attenuation causes a signal larger than the \( e_0 \) output signal to feedback through the capacitor. Therefore, the 1/f curve drops below the unity-gain axis. The circuit remains stable, however, because the intercept point is fixed.

The shape of the 1/f curves indicates the frequencies at which either of the two feedback paths dominate. Feedback is primarily through the \( \beta_C \) block in the flat portion of the 1/f curve labeled \( 1 + \beta_C/R_1 \). Feedback through the \( \beta_C \) block begins at the pole located at \( \omega_p = 20 \pi R_1 C \) and causes the slope of 1/f curve to decrease at 6 dB/octave. Feedback is primarily through \( \beta_C \) block in the flat portion of the 1/f curve labeled \( \epsilon_1/10 \). The curve before this flat portion levels off at the zero located at \( f_z = 1/2 \pi R_1 \parallel R_2 C \), where \( R_1 \parallel R_2 \) is the equivalent parallel resistance of \( R_1 \) and \( R_2 \).

References

Author's biography
Jerry Graeme, a prolific contributor to EDN, manages instrument-components design for Burr-Brown Corp in Tucson, AZ. At Burr-Brown, he has personally designed many analog IC's. He holds a BSEE from the University of Arizona and an MSEE from Stanford.

Article Interest Quotient (Circle One)
High 592 Medium 593 Low 594

Trust Us with Your Eggs

Everyone remembers the old adage about the risk of putting all of one’s eggs in a single basket. That is sage advice, especially in the global age of today with its ever-changing opportunities and hidden pitfalls. Yet this bit of wisdom is not much help in deciding where to put those eggs.

For that, you should ask the experts, and they’ll tell you that the Republic of China on Taiwan is a safe bet for offshore investment. In a recent survey by Business Environment Risk Intelligence (BERI), the ROC had one of the lowest risk ratings for investment of any country in the world, and ranked second only to Japan as the safest place in Asia. That means you can pursue your business goals in the ROC in a trouble-free environment, no matter whether you are a manufacturer using leading-edge technologies or an exporter of fashion blue jeans. Located in the center of the Pacific Rim, the ROC will put you in the heart of Asia, enabling you to utilize the country’s extensive commercial, extensive cultural, and communications links with and communications between the north and south to source the high-tech components from such neighbors as Japan, Korea and raw materials from the Philippines and Indonesia. Further, you will enjoy the benefits of investing in one of Asia’s strongest economies, now bolstered by a US$300 billion Six-Year National Development Plan and by industrial programs that will take the country’s industrial infrastructure to the 21st century.

So where should you put your eggs? The answer is the ROC. For more information, contact the IDIC office nearest you.