Intermodulation at the amplifier-loudspeaker interface

Part 1: Analysis of one source of audible difference between amplifiers

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Intermodulation occurs between an amplified signal and a delayed version returned from a loudspeaker through a feedback loop, when open-loop output impedance is high compared to speaker impedance. Part one of this article analyses this and a second part describes a measurement method with results of tests on different types of amplifier circuit and suggestions for avoiding the effect.

The sound quality at the low-frequency end of the audio reproduction chain has often been discussed in subjective terms as firm, soft, dry and mellow. As far as loudspeakers are concerned, the change in sound impression may be explained as a result of different technical characteristics of the drivers, filters and cabinets. Amplifiers present a more serious problem because the level of harmonic distortion at these frequencies is usually low, the frequency response is relatively flat, and output damping is almost always adequate.

An intriguing question sometimes encountered in practice is why the sound may perceptibly change at the low end of the frequency spectrum when the same listening environment and the same loudspeaker system is used and only the power amplifier is changed. It is our experience that certain power amplifier circuit topologies sound different to others, although no directly explainable difference is noted in the electrical performance of the circuits when tested with resistive load. The following analysis shows that, under certain conditions, the loudspeaker reaction to the drive signal can propagate in the feedback loop of a power amplifier and intermodulate with the drive signal itself. This may partly answer the question.

The dynamic loudspeaker provides a complex load to the amplifier. As much has been written about its behaviour (see, for instance, references 1), it is sufficient here only to present a short list of some of the most important factors affecting the interface between the loudspeaker and the amplifier.

The total compliance of the cone suspension and the loudspeaker cabinet, and the voice coil form a damped mechanical resonance, typically in the frequency range of 30 to 50Hz for the woofer and at correspondingly higher frequencies for the squawker and tweeter. Other mechanical resonances are created by the different moving parts of the cone, excited by the voice coil, but not necessarily rigidly coupled to it. All these mechanical resonances behave like parallel tuned circuits in series with the voice coil resistance and inductance. The crossover filters also exhibit complex reactive behaviour, especially around the crossover frequencies. Figs 1 & 2 show the impedance of two popular loudspeaker systems manifesting both cone and crossover filter resonances.

Energy is stored in all these reactances, especially in the resonances. Because a reaction cannot dissipate energy, and the internal dissipation in the loudspeaker is low at these resonances, most of the stored energy returns to the amplifier and is dissipated in it. In addition, the loudspeaker terminal impedance is non-linear, and cone break-up, delayed responses and acoustical reflections create generator effects in the loudspeaker. Fig. 3 shows a greatly simplified equivalent circuit of a loudspeaker, taking into account only few of the effects discussed.

Now analyse a feedback amplifier having two different loads, as shown in Fig. 4. A pure resistance R is used when measuring the characteristics of the amplifier. A loudspeaker, represented by the grossly simplified equivalent circuit of Fig. 3, is the true load. It is assumed to have a linear resistance R and negligible voice coil inductance L, to facilitate the analysis. The circuit is far from perfect, but this analysis is to illustrate the basic mechanism of distortion only, not to calculate it to a high degree of accuracy. Similarly, the amplifier is assumed to have an infinite input impedance, and no frequency compensation. All these approximations do not affect the result of the analysis. Note that a new parameter, the open-loop output impedance Z, has been incorporated in the circuit in contrast to prior analyses.

The input signal V1 is taken to be a step function V(t), so that its Laplace transform

![Diagram](image)

Fig. 1. Magnitude and phase of the terminal impedance of an Acoustic Research AR3a loudspeaker system, measured with the controls in midposition. Resonant frequencies are 35Hz, 55Hz and 2.5kHz.

![Diagram](image)

Fig. 2. Magnitude and phase of the terminal impedance of a Yamaha NS-1000 Monitor loudspeaker system, measured with the controls in midposition. Resonant frequencies are 55Hz, 410Hz, and 5.5kHz.

![Diagram](image)

Fig. 3. Simplified loudspeaker equivalent circuit. L and C are cone dynamic mass and suspension compliance, respectively. L is the voice coil inductance, R the voice coil resistance, including the radiation resistance, and U is the generator voltage current source.

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The inverse transforms are both perfect step functions and the only difference to standard feedback equations is the term Z/R. An adequate damping factor necessitates that the closed-loop output impedance of the amplifier be much smaller than the loudspeaker impedance, i.e.

$$R \gg Z(1+\beta A)$$

which yields a further simplification. Taking the inverse Laplace transform, the voltages are found in time domain

$$V_d(t) = \frac{A}{1+\beta A} \cdot V_i \cdot u(t)$$

and

$$V_s(t) = \frac{A}{1+\beta A} \cdot V_i \cdot u(0).$$

If now the loudspeaker is substituted for the load, the situation changes markedly. Assuming the damping to be adequate, as by equation 2, equations 1 take the form

$$V_d(s) = \frac{A}{1+\beta A} \cdot |Z(j\omega)|$$

$$\frac{V_d(s)}{V_i(s)} = \frac{Z(1+\beta A)}{1+\beta A} \cdot |Z(j\omega)|$$

and

$$V_s(s) = \frac{A}{1+\beta A} \cdot V_i.$$

No change has occurred in the transformed output voltage $V_s$ of the amplifier. This is to be expected, as the feedback effectively controls the output voltage. However, the internal drive voltage of equation 4 now contains complex terms consisting of the parameters in the loudspeaker equivalent circuit. To study the behaviour of this voltage in time domain, the inverse Laplace transform of equation 4 yields

$$V_d(t) = \frac{A}{1+\beta A} \cdot \left( -Z(j\omega) + \frac{V_i}{1+\beta A} \right) \left( \frac{1}{(R+Z)Q} \exp\left( -\frac{\omega_0^2}{2Q} \right) \sin \omega t \right)$$

where $\omega_0 = \sqrt{(1+\beta A) Z}$, the resonant frequency of the loudspeaker cone, terminals short-circuited, and $Q = v_0/\omega_0$, the approximate quality factor at resonance.

The first term corresponds to the effect of any current generated in the voice coil of the loudspeaker by the vibration of the cone. Assuming that the feedback is large, $1+\beta A \gg 1$, say greater than 30dB, the first term becomes

$$V_d(t) = -Z(j\omega)$$

showing that the amplifier internal drive voltage necessary to serve as a sink for the loudspeaker generator current is directly proportional to the open-loop output impedance $Z$. Dividing this equation by the nominal signal level of equation 3 the ratio of the loudspeaker-generated signal to the driver signal can be found

$$\frac{V_d(t)_{\text{generator}}}{V_d(t)_{\text{signal}}} = \frac{Z}{R+Z} \cdot \frac{V_i(t)_{\text{generator}}}{V_i(t)_{\text{signal}}}$$

Similarly, the last term of equation 5 can be divided by the signal level, equation 3 which yields the ratio of the resonant oscillation in $V_s$ to the signal in $V_d$

$$\frac{V_s(t)_{\text{oscillation}}}{V_s(t)_{\text{signal}}} = \frac{1}{1+\beta A} \cdot \frac{1}{(R+Z)Q} \exp\left( -\frac{\omega_0^2}{2Q} \right) \sin \omega t.$$

This represents a damped oscillation at the cone resonance frequency. There are negative minima and positive maxima at

$$\tau = \frac{1}{\omega_0} \left( \arctan 2Q + \pi n \right)$$

where $n$ is an integer, with values

$$V_d(t) = \frac{Z}{R+Z} \cdot \frac{2}{(1+4Q^2)^{1/2}} \cdot \exp\left( -\frac{\omega_0^2}{2Q} \right) \sin \omega t.$$

Assuming $Z=R$, some typical waveforms of equation 7 are plotted in Fig. 5, and the values of the first minima and maxima are plotted in Fig. 6 as functions of $Q$. The amplitude of oscillation increases with decreasing $Q$. The reason for this apparently strange behavior is that, when the $Q$ of the resonant circuit is lowered, the circuit absorbs more energy from a broadband signal spectrum.
Fig. 7. Measured responses $V_m(t)$ and calculated responses for the AR3a and NS1000M loudspeaker systems. Only the two first resonances around 36Hz and 400Hz were taken into account in the calculated values. The good match of the responses show that the theoretical model used is satisfactory.

To check the validity of the approximations made, the calculated measured responses $V_m(t)$ are shown in Fig. 7 for the two loudspeaker systems of Figs 1 & 2. The calculated results are very close to the measured ones, which is surprising considering the complexity of the real three-way loudspeaker systems. This proves that the simple equivalent circuit of Fig. 3 is satisfactory for this analysis.

To be continued

References
3. Hesser, R., Loudspeaker reviews in various issues of Audio. For review of the Yamaha NS1000, see Audio, January 1979, pp. 82-90.
Intermodulation at the amplifier-loudspeaker interface

Part 2: Causes/how to avoid it/measurements on four types of amplifier circuit

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The effect described is but one of the numerous phenomena affecting the quality of low-frequency sound reproduction. It does not seem probable that its distortion could be dramatically higher than the measured SMPTE - intermodulation distortion of the amplifier, unless protection circuitry malfunctions. However, the theory presented may explain some of the subtle differences in the sound quality between different circuit topologies having otherwise equal standard measurement data. Noting that most valve amplifiers have basically a high open-loop output impedance and employ moderate amounts of feedback (the situation is the inverse for many solid state amplifiers), the theory may also explain some of the audible differences of these amplifiers.

The analysis of part 1 shows that the loudspeaker reflects back to the amplifier signal which may be of the same order of magnitude as the original drive signal. The situation is worse when the open-loop output impedance of the amplifier is comparable to, or greater than, the specified load impedance.

Inside the feedback loop, the amplifier, must now handle two simultaneous large signals - the original drive signal and the loudspeaker reaction signal. If the amplifier has any internal non-linearities, these two signals may interfere and produce intermodulation components with each other. As the input signal is normally composed of a full frequency spectrum, but the loudspeaker-generated reaction consists predominantly of frequency components near the cone resonances and crossover filter resonances, the nature of this distortion is to add coloration to the sound. In addition, the positive maxima shown may cause unwanted clipping near amplifier maximum output power.

The basic reasons for the distortion are that (a) the loudspeaker does not simply consume energy; it stores and returns it. (b) Although the closed-loop output impedance of the amplifier is apparently very low, it is not a true physical impedance as it has been generated by feedback. The feedback, in turn, forces the loudspeaker to react in a manner to cause a corrective signal which circulates around the feedback loop. (c) In the internal non-linearities of the amplifier this signal will intermodulate with the forward signal to produce a change in the spectral composition of the distortion products.

The two basic characteristics affecting the magnitude of this distortion are the open-loop output impedance and the amount of feedback. The dependence is fundamental, i.e. if one or both of these characteristics is brought to zero, interface intermodulation will not occur. The effect increases with feedback if the feedback is small or moderate say, below 20 dB. Above that, increasing feedback will no longer increase distortion. Also, it is generated in the internal non-linearity of the amplifier. As it is basically a low-frequency effect, the stage where the non-linearity is situated in the forward path is immaterial.

The above analysis requires sufficient linearity from the amplifier for the transforms to be valid. In high-quality audio amplifiers this condition is usually met in the normal operating range of the unit. However, a large reaction signal can cause the amplifier to enter a region of severe non-linearity when operated in the vicinity of its maximum output power. The need of a non-linear analysis is indicated in this case.

We propose the following general definition:

Interface intermodulation is a form of distortion in a feedback two-port network, caused by non-linear interaction between the input signal of the two-port and a signal externally injected to the output port propagating into the input via the feedback network.

This general definition is specifically used in sound reproduction equipment to denote the distortion caused by the energy stored or generated in the loudspeaker system re-entering the output of the power amplifier.

Measurement

It is possible to measure interface intermodulation by using normal distortion measurement methods. In this case the standard output loading resistor is replaced with a simulated reactive load or with a real loudspeaker. In many cases the measured distortion is increased and the spectral composition of the distortion products changes. However, in the real-world situation, a set of standardized loudspeaker loads would be needed and, because of the frequency dependencies of these loads, it would be necessary to resort to swept CCIF-type difference tone measurements. This tedious procedure can be replaced by a simpler universal method described below. The loudspeaker reaction can be simulated by letting the amplifier operate on a forward signal, while injecting a backward signal to its output. If interface intermodulation is generated, it will manifest itself through intermodulation products between the two signals appearing at the output.
output. The measuring procedure is thus a
circuit of the two-tone difference-frequency
method. In real life there is a de-
pendence between the forward and back-
ward signals. In this method, these signals are
independent, to facilitate mea-
separation. However, as far as the gener-
tion of intermodulation in the amplifier is
concerned, this does not change the phy-

Fig. 11. Compound-stage circuit (A).
Quiescent current 100 mA, open-loop
output impedance 0.9 ohm.

Fig. 12. Complementary double emitter-
follower circuit configuration (B).
Quiescent current 500 mA, open-loop
output impedance 1.2 ohm.

Fig. 13. Quasi-complementary power
amplifier circuit (C). Quiescent current 100
mA, open-loop output impedance 2.7 ohm.

tically measured.

A proposal for a measurement method is
depicted in Fig. 8. The procedure is:
1. Switch S1 is closed and S2 is open. An
audio-frequency sinusoidal signal is con-
nected to the input of the amplifier un-
der test A1 and is adjusted to yield a
desired output level to a specified load
resistance R1.
2. Amplifier A1 output is disconnected
from the load R1 by opening switch S1.
A low-frequency sinusoidal power
source A2 is connected to the load by
closing switch S2 and is adjusted to yield
the same output level across load R2 as
in step 1. Note: power source A2 has to
have sufficient internal resistance R2 so
as not appreciably change the apparent
load of A2 when switch S2 is closed. This
power source must also have sufficient
output power. A safe rule is that the
rating of the power source is five to ten
times greater than that of the amplifier
under test.
3. Both switches S1 and S2 are closed, with
both output signals being fed simulta-
ecessarily to the load. The intermodula-
tion products between the two signals are
measured across the load by using a
spectrum analyser or an intermodulation
distortion analyser.
4. The r.m.s. sum of all intermodulation
products (i.e. neglecting all harmonic
components of the primary signals) is
calculated and the distortion indicated as
a percentage, referenced to the audio-
frequency signal at the output of A1.
The test frequencies used are in most
cases not critical and can be selected to
minimize the effect of such external dis-

turbances as mains frequency hum. Their
frequency ratio may be optimized so that
the harmonic frequencies of the low-fre-
quency signal do not coincide with the
frequencies of the intermodulation pro-
ducts. Various frequencies and load resis-
tances may be used in different countries,
depending on mains frequency and stan-
dard loudspeaker impedances. The results
reported were obtained using a load resis-
tance of four ohms and frequencies of 63
Hz and 1032 Hz. A typical measurement
result is given in Fig. 9, which shows the
intermodulation spectrum generated.

Comparison of amplifier
circuit topologies

The theory developed predicts that the
amount of intermodulation distortion depends primarily on three basic
power amplifier characteristics: Open-loop
output impedance, amount of feedback,
and closed-loop non-linearity of the cir-
cuit. The first two properties especially
vary considerably among amplifier circuit
topologies. To make a valid overall com-
parison of different topologies, all the cir-
cuits should have

- the same closed-loop gain
- equal closed-loop distortion, and
- same output damping factor, i.e.
closed-loop output impedance.

These rules represent the market place
reality of various commercially competing
amplifier designs having similar overall specifications, irrespective of basic topology. The first rule is based on the assumption that amplifiers of equal output power and equal input sensitivity are compared. The second rule is based on the fact that commercial amplifier designs are limited by a fixed budget. The number of active devices and thus their total gain-distortion quotient is therefore fixed in competing designs of comparable price. Local feedback and overall feedback can then be used in various proportions, but in otherwise optimal designs the total closed-loop intermodulation distortion tends to be the same irrespective of topology, especially at low frequencies which are of interest in the case of interface intermodulation. The third rule is dictated by the commercial necessity of having a reasonable or comparable damping factor specification, irrespective of topology. The circuit shown in Fig. 10 was used for the comparative measurements. Diodes 1 and 2 create an artificial non-linearity, the magnitude of which can be adjusted by changing values of R3, R4, and R5. The same resistors also set the open-loop gain and thereby the amount of overall feedback and damping factor. In the measurements four different output stage configurations were used for the section PA in Fig. 10. Circuits representing popular topologies found in commercial power amplifiers are shown in Fig. 11-14. The operating characteristics of the four circuits to be compared were set up as follows:
- Open-loop gain was increased until the r.m.s. closed-loop output impedance decreased to 0.201.
- Closed-loop total intermodulation distortion was adjusted to 0.2% r.m.s. at an output voltage of 3V pk-pk. By injecting two signals of equal amplitude (63Hz and 1032Hz as in previous case) to the input of the amplifier, the r.m.s. distortion at the output was measured using a resistive 4.1Ω load and referencing the distortion to the 1032Hz signal.
- These two were repeated several times in iterative fashion, as a change in the open-loop non-linearity affected the effective amount of feedback and thereby the output impedance.

In all the measurements, it was made certain that the intrinsic non-linearities of the various output circuits were negligible, as compared to the logarithmic non-linearity of D1, D2 in Fig. 10.

Figure 15 shows the measured closed-loop intermodulation distortion of the various circuits, while Fig. 16 shows the measured open-loop transfer characteristics of the circuits. After adjustment of the circuits, intermodulation measurements were carried out following the procedure outlined earlier. The main results are summarized in the table. Fig. 17 shows the measured values of distortion as functions of the output level. The results are in agreement with the theory presented. They also coincide accurately with earlier results measured for the same circuits using a constant value of feedback in the comparisons.

The results demonstrate clearly the role of the open-loop output impedance of a power amplifier in the generation of interface intermodulation distortion, the various amplifier topologies differing with each other by almost two decades. However, you must not draw far-reaching conclusions of the general usefulness of the various output circuits tested. There may exist ingenious ways to modify any of the topologies so that they will satisfy criteria for low interface distortion. Furthermore, the circuits seem to differ considerably in

Summary of measurement results and conditions

<table>
<thead>
<tr>
<th></th>
<th>A compound</th>
<th>B grounded collector</th>
<th>C quasi-complementary</th>
<th>D grounded emitter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interface distortion at 3V [%]</td>
<td>0.005</td>
<td>0.01</td>
<td>0.1</td>
<td>0.2</td>
</tr>
<tr>
<td>Open-loop output impedance [Ω]</td>
<td>0.9</td>
<td>1.2</td>
<td>2.7</td>
<td>60</td>
</tr>
<tr>
<td>Open-loop gain [dB]</td>
<td>33</td>
<td>36</td>
<td>43</td>
<td>70</td>
</tr>
<tr>
<td>Feedback [dB]</td>
<td>13</td>
<td>16</td>
<td>23</td>
<td>50</td>
</tr>
<tr>
<td>R1 [kΩ]</td>
<td>1</td>
<td>1.5</td>
<td>3.2</td>
<td>1000</td>
</tr>
<tr>
<td>R4 [kΩ]</td>
<td>42</td>
<td>38</td>
<td>32</td>
<td>13</td>
</tr>
</tbody>
</table>

General conditions for circuits: closed-loop gain 20dB; closed-loop output impedance 0.2Ω; closed-loop intermodulation distortion (CCIF) 0.2%; interface intermodulation distortion shown at output level of 3V pk-pk.
their distortion behaviour close to clipping. Although these questions would be of great interest, they are not discussed as the purpose of this article is only to illustrate the basic theory.

The analysis and measurements show:
- a loudspeaker, being reactive by nature, is capable of storing much of the energy it receives from the amplifier.
- this stored energy will be reflected back to the amplifier output terminals.
- the closed-loop output impedance of an amplifier is normally very low, but the open-loop impedance may be several ohms. To damp the reflected signal, feedback will generate a correction signal within the amplifier.
- the signal in the forward path of the amplifier thus consists of two components; the original input signal and the loudspeaker reaction signal, both of the same order of magnitude.
- these two signals may interact in the non-linearities of the amplifier, generating intermodulation products between the two.

- this distortion, termed interface intermodulation, will be most prominent at low frequencies where the loudspeaker reactive load is largest.

### Amplifier design rules to avoid interface intermodulation

The output should provide a low open-loop output impedance to adequately attenuate the loudspeaker reaction signal so that the need for a feedback-generated damping is minimized.

Heavy overall feedback should be applied with caution.

- the susceptibility of the amplifier to interface intermodulation can be measured by using a modified difference-tone method, where one of the signals is injected to the input and one to the output of the amplifier. To create conservative worst-case test for this effect, the latter signal may be increased to equal in power the rated output power of the amplifier.

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